# XXV. *An Account of a Balance of a new Conſtruction, ſuppoſed to be of Uſe in the Woollen Manufacture. By* W. Ludlam, *B. D. Fellow of St.* John's College, Cambridge.

Read June 6, 1765.

IT is of conſequence in ſome branches of the woollen manufacture, that the thread of which any piece is woven ſhould be all of the ſame fineneſs. After it is ſpun, it is made into ſkains of the ſame length, and theſe are ſorted according to the fineneſs of the ſpinning. The manufacturers uſually diſtinguiſh and denominate the fineneſs, by the number of ſkains which go to the pound; the coarſeſt being about 12 to the pound, and the fineſt near 60. There is no other method of ſorting in uſe, except by the eye; but it requires great nicety to diſtinguiſh the ſize of threads ſo ſmall, and long experience to know by the look only, how many ſkains of any particular ſort will make a pound. A method of weighing them readily would ſave much time: the machine here delineated is for that purpoſe. It reſembles the beam of a common pair of ſcales; at one end of it is a fixt weight, which I call the counterpoiſe, at the other a hook: in ſorting, the ſkain to be examined is put upon the hook, and ſinks down more

FIG. 1.

more or leſs, according to its weight, till the counterpoiſe by riſing balances it, and then the index or cock of the beam points out, on a graduated arch, the number of ſkains of that ſort, which go to the pound. A ſcale, inſtead of the hook, might be uſed for weighing money, if the arch were properly divided for that purpoſe.

Mr. Rouſe of Harborough, many years ago, made a machine for ſorting woollen thread upon the ſame principle with this; but as what he did was moſtly tentative, he was not aware of ſome conſiderable advantages which the theory points out. For the machine will not diſtinguiſh with equal nicety the ſkains of every ſize. In Mr. Rouſe's machine, the diviſions were too ſmall, and the largeſt chanced to fall at 18 to the pound; but it would have been better if the finer ſorts had been more accurately diſtinguiſhed, as being of greater conſequence to be well ſorted, and more difficult to be ſorted by the eye only, than the coarſer ones. This machine diſtinguiſhes beſt the yarn of 36 to the pound, one of the fineſt ſorts, as I am informed, in common uſe, the largeſt diviſion lying between 36 and 37; the other diviſions are as large, and the whole range of the index as much as can be allowed without other inconveniencies. The theory contains the neceſſary rules for finding the angle of the beam, for calculating the diviſions on the arch, and for placing their largeſt interval in any part of them.

## *Directions for making the Balance.*

IT conſiſts of a mahogony ſtand, a ſteel beam and braſs ring for the diviſions.

Fig. 1. F G H is the triangular baſe of the ſtand, having a ſcrew in each angle to ſet it level: into this is dove-tail'd the upright back K K; the ſtandard board L L L is put into a dove-tail'd groove in the back, and tenonted into the triangular board at the bottom.

The two cocks C C, between which the arbor of the beam plays, and the ring R R are ſcrew'd to the ſtandard board L L.

The beam A B, with its cock or index E, is broad in the middle to gain ſtrength, and pierced to make it lighter. It is rivetted on a collet ſoldered to the arbor, as clock wheels are. The pivots run in hard braſs, and have plates of hardened ſteel for their points to bear againſt.

At one end of the beam is the counterpoiſe A, conſiſting of two round pieces of braſs, ſcrewed together through an hole in the beam. The other end of the beam is made thicker for about half an inch in length, and ſlit to receive a loop of hardened ſteel, which hangs on the ſteel pin B; on the lower part of the loop hangs the hook, which holds the ſkain.

The block of wood N, is ſcrewed to the ſtandard board; the upper part of which is lined with velvet for the counterpoiſe to reſt upon, when the ſkain is taken off the hook; the braſs pin P is for the other arm of the beam to bank againſt.

The angle of the beam A C B is 168° 6′, the radius C A or C B, 6 inches; the breadth of the beam in the middle $\frac{3}{4}$ of an inch, at the ends $\frac{1}{4}$; the thickneſs at the end B where it is ſlit $\frac{1}{6}$; every where elſe $\frac{1}{18}$. The length of the arbor 1,$\frac{3}{4}$; diameter of the pivets $\frac{1}{30}$. The weight of the counterpoiſe one ounce avoirdupoiſe.

The hook, with the loop and ſteel pin included, 0,68 avoirdupoiſe. The diviſion of 50 is at the top of the ring.

In making the beam, the point of the index E muſt be equally diſtant from the centers A and B; the whole beam and index made ſo as to poiſe itſelf, and remain at reſt in any poſition, before the counterpoiſe ſteel pin and loop are put on.

The counterpoiſe being at firſt made too heavy, to adjuſt it, take off the beam, but let the two cocks and ring remain ſcrewed on; hang then a fine wire and plummet on the top diviſion, and with a watch glaſs look through one pivot hole till you ſee the wire againſt the other; turn the ſcrew G till the wire biſects the hole, and the top diviſion is then perpendicularly over it. Take away the plumb line and put on the beam and counterpoiſe; on the loop hang a weight, which together with the ſteel pin and loop makes one ounce avoirdupoiſe; turn now, by degrees, ſome metal off the counterpoiſe, till the index points at the top diviſion, and the counterpoiſe will be truly adjuſted, whether the arms of the beam, C A, C B, are preciſely equal or not: for though the arms ſhould be nearly equal, it is not neceſſary they ſhould be exactly ſo, as in ſcales.

The diviſions are ſet on the ring by an inſtrument made on purpoſe, which will very readily cut them, tho' unequal, with great exactneſs, on any circle large or ſmall.

To prove the beam, put a weight into the ſcale (that of 28 to the pound is the beſt in this caſe) and ſee if it brings the index down to the proper diviſion on the ring: if it carries it too far, the angle of the beam ACB is too great; if the contrary, too little; and the arms CA, CB, muſt be ſet a little in, or out, till the angle is right: or the angle of the beam may be firſt found experimentally, by the rule hereafter given, and the diviſions calculated to it, which is not much trouble; for having a table ready made for the intended angle, the alterations in that table occaſion'd by a ſmall variation from that angle will be eaſily found by the rules at the end of the following Theory.

When the balance is to be uſed, a weight of $\frac{1}{50}$ of a pound avoirdupoiſe is to be put on the hook: The ſcrew G muſt then be turned, till the index E points at the diviſion of 50: The machine is then properly adjuſted, and the weight may be taken off, and a ſkain put on.

Fig. 2. Let ACB be a bent lever moveable about the angular point C, B a ſcale at the end of the arm B, A a counterpoiſe at the end of the other arm A; Given the angle of the lever, the length of the arms, the reſpective weights of the ſcale and counterpoiſe, to find the poſition of the lever when at reſt.

Produce the arm BC to I, ſo that CI may be to CB as the weight of the ſcale to the counterpoiſe;

join A I, turn the lever about till A I is perpendicular to the horizon, and in that poſition it will be at reſt.

For if H B be drawn parallel to I A, and F C H perpendicular to H B and I A, then C F : C H : C I : C B, or as the ſcale to the counterpoiſe; that is, perpendiculars from the center of motion to the lines of direction are reciprocally as the forces applied, therefore in that caſe the forces balance each other.

Cor. Join A B and draw C G parallel to A I, and A G : G B : : C I : C B, or as the ſcale to the counterpoiſe, therefore G is their center of gravity, which lying in a perpendicular to the horizon paſſing thro' the center of motion, again ſhews the lever to be at reſt.

### SCHOLIUM.

There are indeed two poſitions in which the lever will be at reſt, one when the center of gravity is above, and another when it is below the center of motion; the latter only is conſider'd in what follows.

FIG. 3. Let the arms C A and C B be equal, let the weight of the counterpoiſe be fixt, but that of the ſcale variable; then if the fixt line C A (= C B) repreſent the former, the variable line C I will repreſent the latter. In the line C I, take C D = C A and join A D; thro' the center C, draw E C K parallel to A D, which therefore biſects the angle of the lever A C B, and the part E C may ſtand for the index or cock of the lever. Let C P be perpendicular to the horizon, and E C P the angle of the index with that perpendicular is equal to D A I; but A C K is the ſemi-ſum, and D A I the ſemi-difference of the

angles CAI, CIA: Therefore $CI+CA : CI-CA ::$ or the ſum of the weight of the ſcale and counterpoiſe is to their difference, as the tangent of ACK, half the angle of the lever, to the tangent of DAI =ECP, the inclination of the index to the perpendicular when the lever is at reſt: and the angles at A and I, or the inclinations of the arms, will be the ſum and difference of ACK and DAI.

Hence the ſcale and counterpoiſe being given, the inclination of the index may be found from the angle of the lever; or the angle of the lever from the inclination of the index.

The weight of the counterpoiſe being fixt, let that of the ſcale vary uniformly; it is required to find when the angular motion of the index is greateſt.

All things remaining as before, draw AS perpendicular to CI; and the variation of CI will be the ſame as that of SI (the line CS being conſtant): For a like reaſon the variation of the angle DAI is the ſame with that of SAI. Now the variation of SI, is to that of the arch which meaſures SAI, (AS being radius) as $AI^2$ to $AS^2$; if therefore CI or SI flows uniformly, the fluxion of SAI or DAI, or the angular motion of the index, will be greateſt when AI is leaſt, that is, when AI coincides with AS, or when the inclination of the index is equal to DAS, or when the arm carrying the ſcale is parallel to the horizon.

The angle DAS is the complement of ADS (=ACK) or of half the angle of the lever. Again CA : CS: or radius, to the co-ſine of the angle of the lever as the counterpoiſe to the weight of the ſcale, when the angular motion of the index is greateſt.

Let now the weight of the ſcale as well as the counterpoiſe be fixt, but into it let there be ſucceſſively put variable weights or ſkains of thread, to be denominated by the number of them which (at that ſize) go to the pound: Let the weight of theſe ſkains vary at each ſucceſſive change in ſuch a manner, that the number which goes to the pound may be increaſed each time by unity: It is required to find when the angular variation of the index, in one exchange, will be greateſt.

Fig. 3. All things as before, let CL repreſent the fixt weight of the ſcale, LI the variable one of the ſkain, CI that of both together. Call AS, $s$: SL, $d$: put $p=$ one pound, $x$ the number by which any one ſkain is denominated, conſequently $\frac{p}{x}$ its weight $=$ LI; whence SI$=$LI-LS$=\frac{p}{x}-d$, whoſe fluxion (putting the uniform fluxion of $x$ to be unity) is $\frac{-p}{xx}$: but the fluxion of SI is to that of the arch which meaſures SAI, as $AI^2$ to $AS^2$, or as $AS^2+SI^2$ to $AS^2$, or as $ss+\overline{\frac{p}{x}-d}^{2}:ss$, that is $ss+\overline{\frac{p}{x}-d}^{2}:ss::\frac{-p}{xx}$ to the variation of the index ſought; whence that variation is greateſt when $x=\frac{pd}{ss+dd}$, that is when $\frac{p}{x}$ or the weight of the ſkain $=\frac{ss+dd}{d}$: wherefore if CA repreſent the counterpoiſe, and

Fig. 4. CL the ſcale, join AL and draw AF perpendicular to it, cutting C L produced in F, then will L F repreſent the weight of the ſkain, and D A F be the inclination of the index, when its variation is greateſt.

Fig. 4. On F L as a diameter deſcribe a ſemicircle, and let C A interſect it again in *a*: Join L*a*, *a*F, then will L*a*F be a right angle as well as L A F; therefore with the given angle of the lever A C B and given ſcale C L, we have the ſame maximum F L, either with the counterpoiſe C A, or C*a*; either of which gives the other; for C A : C L :: C F : C*a*.

Fig. 4. In like manner, if the angle of the lever, the counterpoiſe C A, and F L the weight of the ſkain at the maximum are given, with Fig. 5. the center A and radius A E $= \frac{1}{2}$ F L deſcribe a circle cutting B C produced in E and *e*, towards C ſet off E L and *e l*, equal to the radius aforeſaid, and either C L or C*l* will be the weight of the ſcale that will give the propoſed maximum. It is manifeſt that if A E or $\frac{1}{2}$ F L is leſs than A S, the problem is impoſſible; therefore the maximum is limited, and muſt be propoſed ſuch, that the weight of the ſkain in that caſe be leſs than 2 A S.

## Example:

Fig. 4. Let the ſemi-angle of the lever be 84° : 03′, the counterpoiſe one ounce, let the diviſion for 50 to the pound, ſtand at the top of the graduated arch perpendicularly over the center of the lever, that is, let L D $= \frac{16}{50}$ oz. conſequently L C or the ſcale $=$ C D $-$ L D, or C A $-$ L D $= 1 - \frac{16}{50} =$

0,68 oz. then is $CS = 0,9785090$, $LS$ or $d =$ $0,2985090$, $AS$ or $s = 0,2062042$, and $\frac{ss+dd}{d}$ or $\frac{ss}{d} + d$ ($= \frac{AS^2}{LS} + LS$) $= LF = 0,440951$ oz. and $\frac{16}{0.440951} = 36,2852$; therefore the greateſt motion of the index will be when a ſkain of 36 to the pound is changed for one of 37, and the largeſt interval on the graduated arch will be that which lies between 36 and 37.

$CA : CL :: CF : Ca$ or $1 : 0,68 :: 1,120951 : 0,762247$ ounces, the other counterpoiſe which gives the ſame maximum.

Fig. 5. Let the maximum be aſſumed at 36 ſkains to the pound exactly, or when $FL = 0.4444444$, then is $\frac{1}{2} FL$ or $AE = 0.2222222$, and $SE$ ($= \sqrt{\overline{AE + AS} \times \overline{AE - AS}}$) $= 0,0828404$, whence $CL$ ($= \overline{CS - AE} + SE$) $= 0,8391272$, and $CL$ ($= \overline{CS - AE} - SE$) $= 0,6734464$: if this laſt quantity be taken for the weight of the ſcale, the diviſion for 49 in the pound would fall at the top of the arch very nearly, therefore aſſume the diviſion of 50 in the pound for the top, which will make but little alteration in the maximum, and the weight of the ſcale will be 0,68, the maximum between 36 and 37, as before.

$2AS = 0,4124084$, and $\frac{16}{0.4124084} = 48,8419$; therefore with the angle of the lever and counterpoiſe as before, the maximum cannot be higher than at 48 ſkains to the pound. In Mr. Rouſe's machine the angle of the lever was 165°, the counterpoiſe 1.46057 ounces, the ſcale 1,19543 or AS = 0,378024, therefore his machine did not admit of a maximum higher than between 26 and 27 ſkains to the pound.

Fig. 3. To give one example of finding the inclination of the index, let it be required to find the angle for a ſkain of 32 to the pound, whoſe weight is 0. 5 ounce; whence CI = 1.18, CA = 1, and CI + CA : CI − CA; or 2,18 : 0,18 :: Tang. ACK = 84° : 03′. Tang. ECP = 38° : 23′.

This rule for the inclination of the index, ſuggeſts an eaſy way of finding the alterations in it, occaſion'd by any ſmall variation in the angle of the lever, other things remaining the ſame.

For the change in the Logarithmic Tangent of ECP, the fourth term in the proportion, is equal to that made in the Logarithmic Tangent of ACK, the third term; the other two terms being the ſame.

Take therefore the difference of the Logarithmic Tangents of the two ſemi-angles, and divide it by the natural numbers 1, 2, 3, 4, &c. ſucceſſively, and their quotients among the differences of the Logarithmic Tangents, in Tables for each minute, will point out the quantity of the angle ECP, when the alteration in it is 1, 2, 3, 4, &c. minutes ſucceſſively; by which means a table may be eaſily formed for the correction of that firſt computed. Suppoſe, for inſtance, the ſemi-angle of the lever to be only 84°: inſtead of 84° : 03′, for which a table has been made, it is required to correct this table, ſo as to ſuit it to this ſemi-angle 84°. The difference of the Log. Tangents of theſe ſemi-angles is 36.608; divide this by any of the natural numbers, for inſtance by 10, the quotient 3660, ſtanding among the Tabular differences (in Tables for each minute) at 21° : 50′ and 68° : 10′, ſhews that when the value of E C P in the table already made, is either 21°: 50 or 68° : 10′, 10 minutes muſt be ſubtracted to adapt it to the lever whoſe ſemi-angle is only 84° : and ſo of the reſt.

The greateſt difference of the inclination of the index will be when E C P is, 45° : the Tabular differences being then the leaſt. All this is not indeed quite accurate, but ſufficiently ſo for practice.

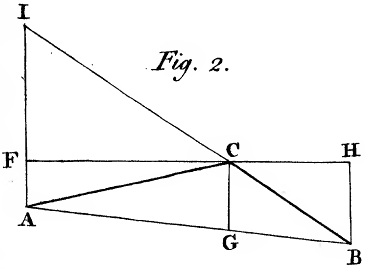
To find accurately when this difference is greateſt, and what that greateſt difference is; add (or ſubtract as the caſe requires) half the difference of the Logarithmic Tangents of the ſemi-angles aforeſaid, to the Logarithmic Radius, and it will give the Logarithmic Tangent of E C P, in that caſe; and the difference between E C P thus found, and its complement, is the greateſt difference ſought. Thus half 36,608 added to 10.0000000 is 10.0018304, the Log. Tangent of 45° : 7′ : 14″,4, the inclination of the index, when its change by reducing the ſemi-angle of the lever to 84°, is greateſt, and the quantity of that change is 14′ : 28″, 8.

A Table of the inclination of the index, when the ſemi-angle of the lever is 84° : 03′, the counterpoiſe 1 ounce and the ſcale 0. 68 ounces avoirdupoiſe, for any number of ſkains to the pound from 10 to 70.

| Skains to the pound. | ° ′ | | ° ′ | | ° | | ° ′ |
|---|---|---|---|---|---|---|---|
| 10 | 75 : 03 | 25 | 52 : 56 | 40 | 20 : 15 | 55 | 8 : 04 |
| 11 | 73 : 56 | 26 | 51 : 00 | 41 | 18 : 02 | 56 | 9 : 30 |
| 12 | 72 : 47 | 27 | 49 : 01 | 42 | 15 : 50 | 57 | 10 : 53 |
| 13 | 71 : 35 | 28 | 46 : 59 | 43 | 13 : 41 | 58 | 12 : 13 |
| 14 | 70 : 20 | 29 | 44 : 54 | 44 | 11 : 35 | 59 | 13 : 30 |
| 15 | 69 : 02 | 30 | 42 : 46 | 45 | 9 : 31 | 60 | 14 : 44 |
| 16 | 67 : 40 | 31 | 40 : 36 | 46 | 7 : 30 | 61 | 15 : 55 |
| 17 | 66 : 16 | 32 | 38 : 23 | 47 | 5 : 32 | 62 | 17 : 03 |
| 18 | 64 : 48 | 33 | 36 : 09 | 48 | 3 : 38 | 63 | 18 : 08 |
| 19 | 63 : 17 | 34 | 33 : 54 | 49 | 1 : 47 | 64 | 19 : 11 |
| 20 | 61 : 42 | 35 | 31 : 37 | 50 | 0 : 00 | 65 | 20 : 12 |
| 21 | 60 : 04 | 36 | 29 : 20 | 51 | 1 : 44 | 66 | 21 : 10 |
| 22 | 58 : 22 | 37 | 27 : 03 | 52 | 3 : 24 | 67 | 22 : 06 |
| 23 | 56 : 37 | 38 | 24 : 46 | 53 | 5 : 01 | 68 | 23 : 00 |
| 24 | 54 : 48 | 39 | 22 : 30 | 54 | 6 : 34 | 69 | 23 : 52 |
| | | | | | | 70 | 24 : 41 |

*N. B.* According to the report from the committee appointed by the Houſe of Commons to enquire into the original ſtandard weights, one pound Avoirdupoiſe is 1 lb. 2 oz. 12 dwt. Troy; conſequently one ounce is 18 dwt. 6 gr. and 0.68 oz. is 12 dwt. 9.84 grains.

*Philos. Trans. Vol. LV. TAB. VI. p. 205.*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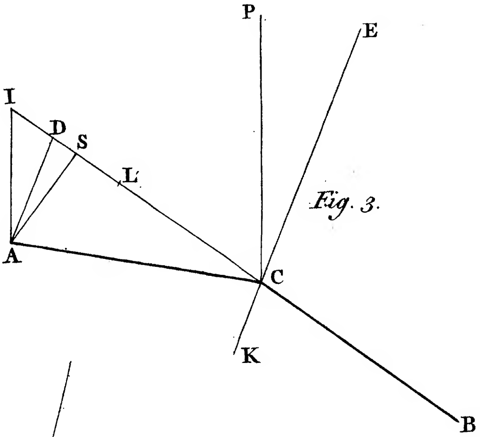
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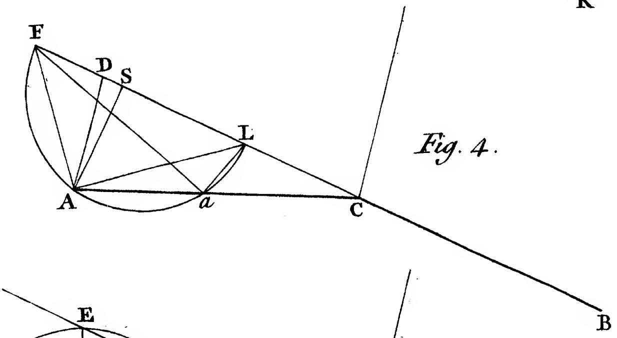
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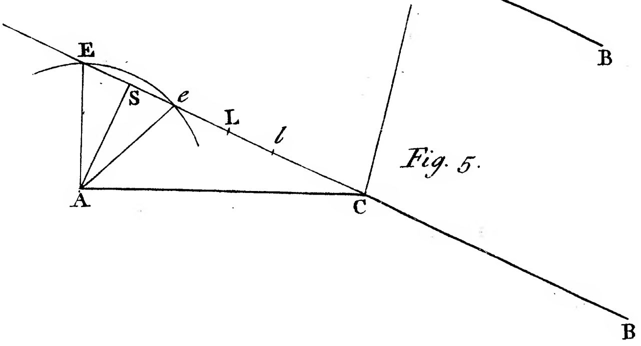
*Fig. 5.*

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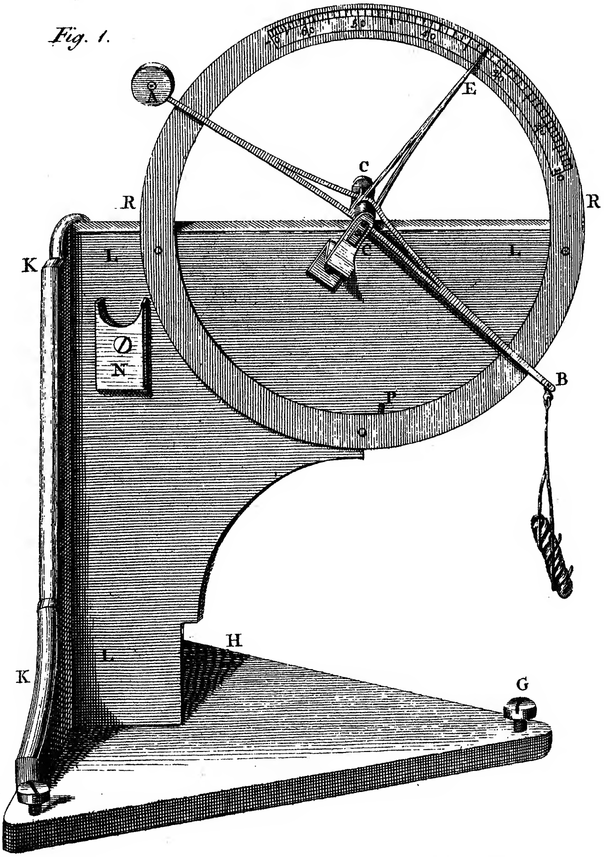
*Philos. Trans. Vol. LV. TAB. V. p. 205.*

*Fig. 1.*



*From a Scale of 3,63 Inches to a Foot.*

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